10-6 Exponential Growth and Decay

Depreciation

$y = a(1 - r)^t$

Decay

$y = ae^{-kt}$

EX 1: A cup of coffee contains 130 milligrams of caffeine. If caffeine is eliminated from the body at a rate of 11% per hour, how long will it take for half of this caffeine to be eliminated from a person's body?

y = new amt

a = original amt

$$y = a(1-r)^t$$
$$65 = 130(1-.11)^t$$
$$\frac{1}{2} = (.89)^t$$
$$\log\frac{1}{2} = t(\log .89)$$
$$\frac{\log\frac{1}{2}}{\log .89} = t$$
$$5.9_{hrs} = t$$

How long will it take for 90% of this caffeine to be eliminated from a person's body?

$$13 = 130(.89)^t$$
$$\frac{1}{10} = .89^t$$
$$\log\left(\frac{1}{10}\right) = t\log(.89)$$
$$\frac{\log\left(\frac{1}{10}\right)}{\log(.89)} = t$$
$$19.8_{hrs} = t$$

EX 2: The half-life of a radioactive substance is the time it takes for half of the atoms of the substance to become disintegrated. All life on Earth contains the radioactive element Carbon-14, which decays continuously at a fixed rate. The half-life of Carbon-14 is 5760 years; that is, every 5760 years, half of a mass of Carbon-14 decays away.

What is the constant for Carbon-14?

$$y = ae^{-kt}$$
$$\frac{1}{2} = e^{-k(5760)}$$
$$\ln\frac{1}{2} = \ln e^{-k(5760)}$$
$$\ln\frac{1}{2} = -5760k$$
$$.00012 = k$$

A paleontologist examining the bones of a wooly mammoth estimates that they contain only 3% as much Carbon-14 as they would have contained when the animal was alive. How long ago did the mammoth die?

$$k = .00012$$
$$y = ae^{-kt}$$
$$3 = 100e^{-.00012t}$$
$$.03 = e^{-.00012t}$$
$$\ln .03 = -.00012t$$
$$29{,}221_{yr} = t$$

EX 3: The half-life of Sodium-22 is 2.6 years. What is the constant for Sodium-22?

$$y = ae^{-kt}$$
$$\frac{1}{2} = 1e^{-k(2.6)}$$
$$\frac{1}{2} = e^{-2.6k}$$
$$\ln\frac{1}{2} = -2.6k$$
$$.2666 = k$$

A geologist examining a meteorite estimates that it contains only about 10% as much Sodium-22 as it would have contained when it reached Earth's surface. How long ago did the meteorite reach the surface of the Earth?

$y = a e^{-.2666t}$

$.10 = 1 e^{-.2666t}$

$.10 = e^{-.2666t}$

$\ln .10 = -.2666t$

$8.6_{yrs} = t$

Appreciation

$y = a\ (1 + r)^t$

$y = a(1 \pm r)^t$

Growth

$y = ae^{kt}$

$y = a e^{\pm kt}$

EX 1: In 1910 the population of a city was 120,000. Since then, the population has increased by exactly 1.5% per year. If the population continues to grow at this rate, what will the population be in the year 2010?

$y = a(1+r)^t$

$y = 120000(1 + .015)^{100}$

$y = 531,845.5$

EX 2: The city of Raleigh, North Carolina grew from a population of 212,000 in 1990 to a population of 259,000 in 1998. Write and exponential growth equation in the form:
$y = ae^{kt}$ where t is the number of years after 1990

$y = ae^{kt}$

Predict the population of Raleigh in 2020.

$259000 = 212000\, e^{k \cdot 8}$

$1.22 = e^{8k}$

$\ln 1.22 = 8k$

$.025 = k$

HW
p563-564
10-19